Name:_____________________________ Date:_______________________________

Math 12 Honours: Section 5.7 Solving Systems with Logarithms

1. Suppose you are given the following solutions to the equation below, which of the solutions are extraneous? Explain:

   $A\log(x+3)+B\log(y+2)=12$
   $C\log(x-2)-D\log(y-4)=-2$
   solutions: $(4,5)$, $(12,18)$, $(1,4)$, $(4,3)$

2. Solve the following systems of equations:

   $3x+4y+5z=29$
   $8x-2y+20z=26$
   $-3x+2y+6z=14$

3. Given the following systems of equations, for what values of "C" will there be an infinite number of solutions? Explain. For what value(s) of "C" will there be 1 solution OR No solutions. Explain:

   $3x+4y=12$
   $6x+8y=C$

4. Given that the systems of equations have one solution, what are the values of "B" and "C":

   $2x+5y=12$
   $7x+By+C=0$

5. Given each systems of equation, find all sets of real numbers (x, y) that satisfies it:

$\log_2 x+\log_2 y=9$ $\log_4 x+4\log_2 y=22$	$5\log_3 x+6\log_3 y=14$ $4\log_3 x-3\log_3 y=6$
$\log_x 4+\log_y \sqrt{3}=5$ $\log_x 8-\log_y 9=13$	$(3x)^{\log 3}=(6y)^{\log 6}$ $6^{\log x}=3^{\log y}$

$\log_8 4x - \log_8 y = \frac{4}{3}$ $4^{\log_4(4x-y)} = \log_2 16$	$28y^4 = x^2 + 3$ $\log_x y^2 = \log_{y^2} x$
$\log x - \log 3y = 1$ $3^{3x+y} = 27$	Solve the following system of equation: $\log x^3 + \log y^2 = 11$ and $\log x^2 - \log y^3 = 3$ Euclid 2003

6. Solve the system of equations: $\begin{matrix} \log x^3 + \log y^2 = 11 \\ \log x^2 - \log y^3 = 3 \end{matrix}$ [Euclid]

7. Determine all the triples (x,y,z) of real numbers that are solutions to the following system of equations [Euclid]:

$$\log_9 x + \log_9 y + \log_3 z = 2$$
$$\log_{16} x + \log_4 y + \log_{16} z = 1$$
$$\log_5 x + \log_{25} y + \log_{25} z = 0$$

8. Determine all real solutions to the system of equations and prove that there are no more solutions: Euclid 2008

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\begin{aligned}
x+\log(x) &= y-1 \\
y+\log(y-1) &= z-1 \\
z+\log(z-2) &= x+2
\end{aligned}
$$

9. Determine all real values of "x" for which the equation is true: [Euclid]

$\log_{225} x+\log_{64} y=4$ & $\log_{x} 225-\log_{y} 64=1$

10. $\sqrt{\log_2 x\cdot\log_2(4x)+1}+\sqrt{\log_2 x\cdot\log_2\left(\frac{x}{64}\right)+9}=4$ Solve the following system of equations: Note There are two solutions: AMC 2002

11. Let: $\begin{array}{l} S_1=\{(x,y)\,|\,\log(1+x^2+y^2)\le 1+\log(x+y)\} \\ S_2=\{(x,y)\,|\,\log(2+x^2+y^2)\le 2+\log(x+y)\} \end{array}$. What is the ratio of the area of S_2 to the area of S_1? 2006 AMC 12A

12. Consider the following system of equations, solve the system, (a,b,c) = (-4, 4, -18):

$$(\log x)(\log y) - 3\log 5y - \log 8x = a$$
$$(\log y)(\log z) - 4\log 5y - \log 16z = b$$
$$(\log z)(\log x) - 4\log 8x - 3\log 625z = c$$

b) Determine all triples (a,b,c) of real numbers for which the system of equations has an infinite number of solutions (x, y, z)

13. Let “S” be the set of ordered triples (x,y,z) of real numbers for which:

$\log(x+y) = z$ and $\log(x^2 + y^2) = z+1$.

There are real numbers ‘a” and “b” such that for all ordered triples (x,y,z) in “S” have $x^3 + y^3 = a\times 10^{3z} + b\times 10^{2z}$. What is the value of $a+b$? AMC 2005 12B